Green dyadics in reciprocal uniaxial bianisotropic media by cylindrical vector wave functions

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The reciprocal uniaxial bianisotropic medium, which can be fabricated by polymer synthesis techniques, is a generalization of the well-studied chiral medium. It has potential applications in the design of antireflection coating, antenna radomes, and interesting microwave components. In the present investigation, based on the concept of spectral eigenwaves, eigenfunction expansion of the Green dyadics in this class of materials is formulated in terms of cylindrical vector wave functions. The formulations are greatly simplified by analytically evaluating the integrals with respect to the spectral longitudinal and radial wave numbers, respectively. The analysis indicates that the solutions of the source-incorporated Maxwell's equations for a homogeneous reciprocal uniaxial bianisotropic medium are composed of two eigenwaves traveling with different wave numbers, and each of these eigenwaves is a superposition of two transverse waves and a longitudinal wave. The Green dyadics of planarly and cylindrically multilayered structures consisting of the reciprocal uniaxial bianisotropic media can be straightforwardly obtained by applying the method of scattering superposition and appropriate electromagnetic boundary conditions, respectively. The resulting formulations, which can be theoretically verified by comparing their special forms with existing results, provide a fundamental basis to analyze and understand the physical phenomena of unbounded and multilayered reciprocal uniaxial bianisotropic media. The method employed here can be generalized to derive the eigenfunction expansion of Green dyadics in other kinds of media. [S1063-651X(96)10208-7]

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I. INTRODUCTION

The concept of vector wave functions was first proposed by Hansen [1] to solve the source-free Maxwell's equations in isotropic media. This vector-wave-function approach has been intensively developed by Felsen and Marcuvitz [2], Morse and Feshbach [3], and Tai [4], to investigate the source-incorporated electromagnetic boundary value phenomena of isotropic media. It has been discovered that for some types of electromagnetic boundary value problems of isotropic media (e.g., microstrip wraparound antennas [5], circular-shaped microwave radiators [6,7], and excitations of cylindrical waveguides and cavities [8]), field representations and Green dyadics by the cylindrical vector wave functions are more useful than those by the planar vector wave functions. Recently, field representations by the cylindrical vector wave functions of isotropic media were presented for the source-free gyroelectric chiral media [9], composite chiral-ferrite media [10], reciprocal uniaxial bianisotropic media [11], and uniaxial bianisotropic-ferrite media [12]. However, analytical solutions to the source-incorporated Maxwell's equations in any given complex media still need to be developed, so as to provide methodological convenience in studying the physical phenomena of these materials.

The Green dyadic is one of the basic tools that are used to solve the source-incorporated Maxwell's equations. It is useful both in analyzing radiation problems [4,13] and in constructing integral equations for scattering phenomena [14,15]. The general representation of the Green dyadic expressed in terms of an expansion of the vector wave functions are required to study Raman and fluorescent scattering by active molecules embedded in a particle [16,17], as well as to establish the T-matrix formulation from Huygens's principle and extinction theorem [18,19]. Furthermore, eigenfunction expansion of the Green dyadics could also provide fundamental insight into the physical process of the material under consideration. However, much effort is still required in order to obtain the Green dyadics in any given complex media when expressed in the full eigenfunction expansion of the vector wave functions.

With recent development of polymer synthesis techniques [11], increasing attention is being attracted to the analysis of interaction of electromagnetic waves with interesting microwave materials, in order to determine how to use these materials to provide better solutions to current engineering problems. Among these microwave materials, one should mention the reciprocal uniaxial bianisotropic medium [11], because of its potential applications in microwave technology, antenna design, and particularly in antireflection coating. In practice, a reciprocal uniaxial bianisotropic medium with linear magnetoelectric interaction can be fabricated by arranging two types of microstructures (short helices and Ω-shaped elements) in the same isotropic host material. From a phenomenological point of view, a homogeneous reciprocal uniaxial bianisotropic medium can be characterized by the set of constitutive relations [11]

$$\mathbf{D}=\overline{\overline{\boldsymbol{\epsilon}}}\cdot\mathbf{E}+\overline{\overline{\boldsymbol{\xi}}}\cdot\mathbf{H}, \tag{1a}$$

$$\mathbf{B}=\overline{\boldsymbol{\mu}}\cdot\mathbf{H}+\overline{\boldsymbol{\zeta}}\cdot\mathbf{E}, \tag{1b}$$

where

$$\overline{\boldsymbol{\varepsilon}}=\varepsilon_t\overline{I}_t+\varepsilon_z\mathbf{e}_z\mathbf{e}_z, \tag{2a}$$

$$\overline{\boldsymbol{\mu}}=\mu_t\overline{\mathbf{I}}_t+\mu_z\mathbf{e}_z\mathbf{e}_z \tag{2b}$$

are the permittivity and permeability dyadics, respectively, and

$$\overline{\boldsymbol{\xi}}=i(\mu_0\varepsilon_0)^{1/2}(-\alpha\overline{\mathbf{I}}_t+\beta\mathbf{e}_z\times\overline{\mathbf{I}}_t-\gamma\mathbf{e}_z\mathbf{e}_z) \tag{2c}$$

and

$$\overline{\boldsymbol{\zeta}}=i(\boldsymbol{\mu}_0\boldsymbol{\varepsilon}_0)^{1/2}(\alpha\overline{\mathbf{I}}_t+\beta\mathbf{e}_z\times\overline{\mathbf{I}}_t+\gamma\mathbf{e}_z\mathbf{e}_z) \tag{2d}$$

denote the magnetoelectric pseudodyadics. Here, $\overline{\mathbf{I}}_t=\mathbf{e}_x\mathbf{e}_x+\mathbf{e}_y\mathbf{e}_y$ stands for the transverse unit dyadic, and $\mathbf{e}_j$ represents the unit vector in the j direction. Instead of three constitutive parameters for the well-studied chiral media [20,21], we are facing a medium with seven scalar parameters. It is apparent that the constitutive dyadics of the medium satisfy the reciprocal conditions [22] as well as the uniformity constraint condition [23]. For a lossless reciprocal uniaxial bianisotropic medium, the scalar constitutive parameters ε_t, ε_z, μ_t, μ_z, α, β, and γ are all real, which are assumed throughout the present consideration.

From the view point of reciprocal conditions [22], the constitutive relations of (1a) and (1b) are the most general forms of uniaxial materials which satisfy the requirements of reciprocal theorem. To get an idea of a medium with constitutive dyadics of the above forms, we first note that the special case of $\beta=\gamma=0$ corresponds to the transversely chiral uniaxial bianisotropic medium studied earlier [24]. This medium can be created by suspending metal helices in an isotropic host material in such a way that the axes of all helices are perpendicular to the z axis, but possess arbitrary orientations and locations. In another special case with $\alpha=\gamma=0$, the present medium becomes the uniaxial omega medium [25], which may be fabricated by immersing two ensembles of orthogonally positioned Ω-shaped particles in an isotropic host medium. When $\alpha=\beta=0$, the medium is called a uniaxial chiral medium [26], which can be realized by mixing conductive helices in an isotropic host medium in such a manner that the axes of all helices are parallel to the z axis but with random locations. The medium under consideration reduces to a uniaxial chiro-omega medium, as γ vanishes [27]. A uniaxial chiro-omega medium, fabricated by immersing both metal helices and Ω-shaped elements in the same isotropic host medium in a certain manner, may find applications in the design of antireflection coating and antenna radomes.

The reciprocal uniaxial bianisotropic medium is a subset of the wider class referred to as bianisotropic media. Important research on general bianisotropic media have been presented by Post [28], Kong [22], and Chen [29], among others. In contradistinction to these general considerations, the present contribution is intended to derive the eigenfunction expansion of the Green dyadics in a homogeneous reciprocal uniaxial bianisotropic medium in terms of the cylindrical vector wave functions. Based on the completeness property of the spectral eigenwaves in the Fourier transformation spectral domain, the present formulations are considerably simplified by analytically evaluating the integrals with respect to the spectral longitudinal and radial wave numbers, respectively. This extended method, which is standard and straightforward, leads to two sets of the eigenfunction expansion of the Green dyadics in terms of the cylindrical vector wave functions. The analysis indicates that the solutions of the source-incorporated Maxwell's equations in a reciprocal uniaxial bianisotropic medium are composed of two eigenwaves traveling with different wave numbers, and each of these eigenwaves is a superposition of two transverse waves and a longitudinal wave. It is also found that the Sommerfeld-Weyl-type integrals of dipole radiation in a reciprocal uniaxial bianisotropic medium involve only those Sommerfeld-Weyl-type integrals of dipole radiation in an isotropic medium. The present formulations can be used to construct the Green dyadics of planarly and cylindrically multilayered structures consisting of the reciprocal uniaxial bianisotropic media, by employing the method of scattering superposition [4,20,21] and appropriate electromagnetic boundary conditions, respectively. The greatest advantage of the Green dyadics, which are represented in the forms of the eigenfunction expansion, is that they provide a fundamental insight into the physical process of the reciprocal uniaxial bianisotropic medium, and lay the theoretical foundation to study the source-incorporated electromagnetic phenomena involving the reciprocal uniaxial bianisotropic media (e.g., Raman and fluorescent scattering by active molecules embedded in a reciprocal uniaxial bianisotropic medium).

A closed-form expression of the Green dyadic for a special class of uniaxial bianisotropic media with $\alpha=\beta=0$, was first derived for the reciprocal case [30], and later for the nonreciprocal case $\mathbf{e}_z\cdot\overline{\boldsymbol{\xi}}\cdot\mathbf{e}_z\neq-\mathbf{e}_z\cdot\overline{\boldsymbol{\zeta}}\cdot\mathbf{e}_z$ [31]. In [32], a rigorous investigation was presented by Weiglhofer for the possibility of deriving the closed-form representations of the Green dyadics in a general uniaxial media. In that paper, it was shown that at least one of three possible relations among the constitutive parameters has to be satisfied to allow the closed-form solutions of the Green dyadics. It was also pointed out, however, that these relations are only necessary relations and not sufficient relations to allow the closed-form solution. The most important of these three cases is the case with $\beta=\gamma=0$, for which the closed-form solutions of the Green dyadics were presented in [32]. Most recently, Olyslager [33] presented closed-form representations of the Green dyadics for a uniaxial bianisotropic media with $\beta=0$. In view of the uniformity constraint condition for the uniaxial bianisotropic media [23], the materials treated in [30–33] are just special cases of the media studied here. The methods used by the authors of [30–33] do not seem applicable for the present most general reciprocal uniaxial bianisotropic media to allow the closed-form representations of the Green dyadics. Moreover, the Green dyadics represented in the forms of the eigenfunction expansion seem to be more important and attractive than those expressed in the closed forms in practical applications (e.g., to study quantitatively the Raman and fluorescent scattering by active molecules embedded in the given complex media, to establish the T-matrix formulation for the electromagnetic boundary value problems involving complex media, and to qualitatively take insight into the physical precess of the material under consideration). In the following analysis, the harmonic $\exp(i\omega t)$ time dependence is assumed and suppressed throughout.

II. EIGENWAVES IN RECIPROCAL UNIAXIAL BIANISOTROPIC MEDIUM

Substituting the constitutive relations (1a) and (1b) into the source-incorporated Maxwell's equations, a compact form of the field equations in the reciprocal uniaxial bianisotropic medium is obtained:

$$\begin{bmatrix} \omega\bar{\bar{\varepsilon}}\cdot\bar{\bar{I}} & \omega\bar{\bar{\xi}}\cdot\bar{\bar{I}}+i\nabla\bar{\bar{I}} \\ \omega\bar{\bar{\zeta}}\cdot\bar{\bar{I}}-i\nabla\bar{\bar{I}} & \omega\bar{\bar{\mu}}\cdot\bar{\bar{I}} \end{bmatrix}\begin{bmatrix}\mathbf{E}(\mathbf{r})\\ \mathbf{H}(\mathbf{r})\end{bmatrix}=\begin{bmatrix} i\mathbf{J}_e(\mathbf{r})\\ i\mathbf{J}_m(\mathbf{r})\end{bmatrix}, \tag{3}$$

where $\mathbf{J}_e$ and $\mathbf{J}_m$ denote the electric and magnetic exciting currents, respectively.

To examine the physical properties of the eigenwaves in the reciprocal uniaxial bianisotropic medium, Fourier transformation for the electromagnetic fields and exciting sources is introduced:

$$\mathbf{F}(\mathbf{r})=\frac{1}{8\pi^3}\int_{-\infty}^{\infty}\mathbf{F}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{r}}d\mathbf{k}, \tag{4}$$

where $\mathbf{F}=\mathbf{E}$, H, $\mathbf{J}_e$, and $\mathbf{J}_m$, and $\mathbf{k}=k_x\mathbf{e}_x+k_y\mathbf{e}_y+k_z\mathbf{e}_z$. Then (3) can be rewritten in the Fourier spectral domain

$$\begin{bmatrix} \omega\bar{\bar{\varepsilon}}\cdot\bar{\bar{I}} & \omega\bar{\bar{\xi}}\cdot\bar{\bar{I}}+\mathbf{k}\bar{\bar{I}} \\ \omega\bar{\bar{\zeta}}\cdot\bar{\bar{I}}-\mathbf{k}\bar{\bar{I}} & \omega\bar{\bar{\mu}}\cdot\bar{\bar{I}} \end{bmatrix}\begin{bmatrix}\mathbf{E}(\mathbf{k})\\ \mathbf{H}(\mathbf{k})\end{bmatrix}=\begin{bmatrix} i\mathbf{J}_e(\mathbf{k})\\ i\mathbf{J}_m(\mathbf{k})\end{bmatrix}. \tag{5}$$

For the sake of brevity, Eq. (5) is rewritten as

$$\bar{\bar{\mathbf{L}}}\cdot\mathbf{\Psi}(\mathbf{k})=\mathbf{\Phi}(\mathbf{k}), \tag{6}$$

where $\bar{\bar{\mathbf{L}}}$ is a Hermitian operator (i.e., $\bar{\bar{\mathbf{L}}}=\bar{\bar{\mathbf{L}}}^{*\mathrm{T}}$, where the superscripts asterisk and T denote complex conjugate and transpose, respectively). Here $\mathbf{\Psi}(\mathbf{k})=[\mathbf{E}(\mathbf{k}),\mathbf{H}(\mathbf{k})]^T$, and $\mathbf{\Phi}(\mathbf{k})=[i\mathbf{J}_e(\mathbf{k}),i\mathbf{J}_m(\mathbf{k})]^T$.

The characteristic equation, which determines the wave numbers of the eigenwaves propagating in the reciprocal uniaxial bianisotropic medium, can be straightforwardly obtained by requiring the determinant of operator $\mathbf{L}$ be zero. Algebraic manipulation results in

$$\varepsilon'(f^2-a)k_\rho^4+[(k_z^2-a)(e^2+f^2-a-\varepsilon'a')+b(b-2e)k_z^2]k_\rho^2-[(k_z^2-a)^2+b^2k_z^2]a'=0, \tag{7}$$

where $k_\rho=(k_x^2+k_y^2)^{1/2}$, and

$$\begin{aligned} a&=\omega^2[\varepsilon_t\mu_t-\varepsilon_0\mu_0(\alpha^2+\beta^2)],\\ b&=2ik_0\alpha,\\ e&=ik_0(\alpha+\gamma\varepsilon'),\\ f&=ik_0\beta,\\ \varepsilon'&=\varepsilon_t/\varepsilon_z,\\ a'&=\omega^2(\varepsilon_t\mu_z-\varepsilon_0\mu_0\gamma^2\varepsilon'). \end{aligned} \tag{8}$$

It is obvious that the characteristic equation (7) is an even function of k_ρ (or k_z). We can regard this characteristic equation (7) as a function of k_ρ (or k_z), where k_ρ (or k_z) is determined by k_z (or k_ρ). The roots of Eq. (7) are designated as $k_\rho=k_{\rho q}$ (or $k_z=k_{zq}$), where $q=1$, 2, 3, and 4. It is worthy noting the important property of the roots of Eq. (7): $k_{\rho q}$ (or k_{zq}) is independent of ϕ_k, with $\phi_k=tg^{-1}(k_y/k_x)$.

The eigenwaves corresponding to the qth root of (7), expressed in the circular cylindrical coordinate system, can be derived by substituting $k_\rho=k_{\rho q}$ or $k_z=k_{zq}$ in the following expression:

$$\mathbf{\Psi}_q^\sigma(\mathbf{k})=\begin{bmatrix} E_{q\rho}^\sigma(\mathbf{k})\\ E_{q\phi}^\sigma(\mathbf{k})\\ E_{qz}^\sigma(\mathbf{k})\\ H_{q\rho}^\sigma(\mathbf{k})\\ H_{q\phi}^\sigma(\mathbf{k})\\ H_{qz}^\sigma(\mathbf{k})\end{bmatrix}=\begin{bmatrix} C_q^\sigma(k_\rho,k_z)\cos(\phi-\phi_k)+D_q^\sigma(k_\rho,k_z)\sin(\phi-\phi_k)\\ -C_q^\sigma(k_\rho,k_z)\sin(\phi-\phi_k)+D_q^\sigma(k_\rho,k_z)\cos(\phi-\phi_k)\\ \frac{1}{\omega\varepsilon_z}[ik_0\gamma-k_\rho B_q^\sigma(k_\rho,k_z)]\\ A_q^\sigma(k_\rho,k_z)\cos(\phi-\phi_k)+B_q^\sigma(k_\rho,k_z)\sin(\phi-\phi_k)\\ -A_q^\sigma(k_\rho,k_z)\sin(\phi-\phi_k)+B_q^\sigma(k_\rho,k_z)\cos(\phi-\phi_k)\\ 1\end{bmatrix}, \tag{9}$$

with $\phi=tg^{-1}(y/x)$ and $\sigma=\rho$ for $k_\rho=k_{\rho q}$, and $\sigma=z$ for $k_z=k_{zq}$. Here the spectral parameters are found to be

$$A_q^\sigma(k_\rho,k_z)=k_\rho[(k_z-f)(\varepsilon'k_\rho^2+k_z^2-a)+bek_z]/E_q^\sigma(k_\rho,k_z), \tag{10}$$

$$B_q^\sigma(k_\rho,k_z)=k_\rho[e(k_z^2-a)-bk_z(k_z-f)]/E_q^\sigma(k_\rho,k_z), \tag{11}$$

$$E_q^\sigma(k_\rho,k_z)=(k_z^2-a)(\varepsilon'k_\rho^2+k_z^2-a)+b^2k_z^2, \tag{12}$$

$$C_q^\sigma(k_\rho,k_z)=\frac{1}{\omega\varepsilon_t}[ik_0\alpha A_q^\sigma(k_\rho,k_z)+(ik_0\beta+k_z)B_q^\sigma(k_\rho,k_z)], \tag{13}$$

and

$$D_q^\sigma(k_\rho,k_z)=\frac{1}{\omega\varepsilon_t}[ik_0\alpha A_q^\sigma(k_\rho,k_z)+k_\rho-(ik_0\beta+k_z)A_q^\sigma(k_\rho,k_z)]. \quad (14)$$

To reveal the biorthogonality property of these eigenwaves, Eq. (6) should be rewritten in other forms. First, regarding $k_{\rho q}$ as the roots of the characteristic equation (7), (6) is rewritten as

$$\overline{\mathbf{A}}_1\cdot\mathbf{\Psi}_q^\rho(\mathbf{k})-k_{\rho q}\overline{\mathbf{B}}_1\cdot\mathbf{\Psi}_q^\rho(\mathbf{k})=\mathbf{\Phi}(\mathbf{k}), \quad (15)$$

where both $\overline{\mathbf{A}}_1$ and $\overline{\mathbf{B}}_1$ are Hermitian operators. These eigenwaves $\mathbf{\Psi}_q^\rho(\mathbf{k})$, which form a complete set in the spectral space [29], are biorthogonality [29,34]: $\mathbf{\Psi}_p^{\rho*}(\mathbf{k})\cdot\mathbf{B}_1\cdot\mathbf{\Psi}_q^\rho(\mathbf{k})=N_p^2\delta_{pq}$. Here δ_{pq} denotes the Kronecker delta function (i.e., it is 1 for $p=q$, and 0 for $p\neq q$).

An alternative useful rewritten form of (6) is

$$\overline{\mathbf{A}}_2\cdot\mathbf{\Psi}_q^z(\mathbf{k})-k_{zq}\overline{\mathbf{B}}_2\cdot\mathbf{\Psi}_q^z(\mathbf{k})=\mathbf{\Phi}(\mathbf{k}), \quad (16)$$

where both $\overline{\mathbf{A}}_2$ and $\overline{\mathbf{B}}_2$ are Hermitian operators, and the roots of the characteristic Eq. (7) are considered to be k_{zq}. The eigenwaves of Eq. (16), which form a complete set in the spectral space [29], are also biorthogonality [29,34]:

$$\mathbf{\Psi}_p^{z*}(\mathbf{k})\cdot\overline{\mathbf{B}}_2\cdot\mathbf{\Psi}_q^z(\mathbf{k})=M_p^2\delta_{pq}.$$

Based on the completeness properties of the above-presented eigenwaves $\mathbf{\Psi}_q^z(\mathbf{k})$ and $\mathbf{\Psi}_q^\rho(\mathbf{k})$, the solutions of the spectral source-incorporated equation (6) can be represented in terms of these eigenwaves [2,29,34]:

$$\mathbf{\Psi}(\mathbf{k})=\sum_q\frac{\mathbf{\Psi}_q^z(\mathbf{k})\mathbf{\Psi}_q^{z*}(\mathbf{k})}{(k_{zq}-k_z)M_q^2}\cdot\mathbf{\Phi}(\mathbf{k}) \quad (17)$$

or

$$\mathbf{\Psi}(\mathbf{k})=\sum_q\frac{\mathbf{\Psi}_q^\rho(\mathbf{k})\mathbf{\Psi}_q^{\rho*}(\mathbf{k})}{(k_{\rho q}-k_\rho)N_q^2}\cdot\mathbf{\Phi}(\mathbf{k}). \quad (18)$$

In this way, the solutions of the spectral source-incorporated Maxwell's equation (5) are represented in terms of the spectral eigenwaves in the reciprocal uniaxial bianisotropic medium. These expressions, (17) and (18), are our starting point in constructing the eigenfunction expansion of the Green dyadics, as will be reported in detail in the following analysis.

III. GREEN DYADICS IN RECIPROCAL UNIAXIAL BIANISOTROPIC MEDIUM

For the sake of simplicity, we define the Green dyadics $\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')$ in the homogeneous reciprocal uniaxial bianisotropic medium as

$$\begin{bmatrix}\mathbf{E}(\mathbf{r})\\ \mathbf{H}(\mathbf{r})\end{bmatrix}=\int_{V'}\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')\cdot\begin{bmatrix}i\mathbf{J}_e(\mathbf{r}')\\ i\mathbf{J}_m(\mathbf{r}')\end{bmatrix}dV', \quad (19)$$

where V' is the volume occupied by the electric and magnetic exciting currents. Definition (19) indicates that the electromagnetic fields associated with the current sources can be expressed as a convolution of the current distribution and the three-dimensional free-space Green dyadics.

Using the definition of Green dyadics (19) and Eqs. (17) and (18), the spatial Green dyadics in the reciprocal uniaxial bianisotropic medium can be represented in terms of the corresponding spectral eigenwaves

$$\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')=\frac{1}{8\pi^3}\int_{-\infty}^{\infty}d\mathbf{k}\sum_q\frac{\mathbf{\Psi}_q^z(\mathbf{k})\mathbf{\Psi}_q^{z*}(\mathbf{k})}{(k_{zq}-k_z)M_q^2}e^{-i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}, \quad (20)$$

$$\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')=\frac{1}{8\pi^3}\int_{-\infty}^{\infty}d\mathbf{k}\sum_q\frac{\mathbf{\Psi}_q^\rho(\mathbf{k})\mathbf{\Psi}_q^{\rho*}(\mathbf{k})}{(k_{\rho q}-k_\rho)N_q^2}e^{-i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}. \quad (21)$$

Here the convolution theorem of Fourier transformation has been employed.

It is helpful to mention that Eq. (20) is suitable to construct the Green dyadics of planarly multilayered reciprocal uniaxial bianisotropic media, while Eq. (21) is a useful tool to formulate the Green dyadics of a cylindrically multilayered structure consisting of the reciprocal uniaxial bianisotropic media. To represent the Green dyadics in the forms of the eigenfunction expansion in terms of the cylindrical vector wave functions, integrals with respect to the spectral longitudinal and radial wave numbers in Eqs. (20) and (21), respectively, will be evaluated analytically.

A. Analytical evaluation of the integral with respect to the spectral longitudinal wave number

In this subsection, we will try to represent (20) in the form of the eigenfunction expansion in terms of the cylindrical vector wave functions. For this purpose, the integral with respect to the spectral longitudinal wave number k_z is analytically evaluated by using the residue method, which results in

$$\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')=\begin{bmatrix}\overline{\mathbf{G}}_{ee}(\mathbf{r},\mathbf{r}')\overline{\mathbf{G}}_{em}(\mathbf{r},\mathbf{r}')\\ \overline{\mathbf{G}}_{me}(\mathbf{r},\mathbf{r}')\overline{\mathbf{G}}_{mm}(\mathbf{r},\mathbf{r}')\end{bmatrix}=\frac{i}{8\pi^2}\int_0^\infty dk_\rho\int_{\phi_k=0}^{2\pi}d\phi_k\sum_{q=1}^{2}\frac{\mathbf{\Psi}_q^z(\mathbf{k})\mathbf{\Psi}_q^{z*}(\mathbf{k})}{M_q^2}\times e^{-ik_{zq}(z-z')}e^{-ik_\rho\rho\cos(\phi-\phi_k)}e^{ik_\rho\rho'\cos(\phi'-\phi_k)}, \quad (22)$$

with $\rho=(x^2+y^2)^{1/2}$. Here the 3×3 dyadics $\overline{\mathbf{G}}_{ee}(\mathbf{r},\mathbf{r}')$ and $\overline{\mathbf{G}}_{mm}(\mathbf{r},\mathbf{r}')$ are the Green dyadics of electric and magnetic types, respectively, while $\overline{\mathbf{G}}_{em}(\mathbf{r},\mathbf{r}')$ and $\overline{\mathbf{G}}_{me}(\mathbf{r},\mathbf{r}')$ are the pseudotype Green dyadics. Noting $k_{z3}=-k_{z1}$ and $k_{z4}=-k_{z2}$, the symmetric roots k_{z3} and k_{z4} are not included in the summation of Eq. (22), since these symmetric roots are automatically taken into account as the spectral azimuthal angle ϕ_k spans from 0 to 2π. It should be recognized that the following formulations are essentially based on the fact that the spectral longitudinal wave number k_{zq} is independent of the spectral azimuthal angle ϕ_k.

Substituting into Eq. (22) the explicit expression of $\mathbf{\Psi}_q^z(\mathbf{k})$ and the well-known identities

$$e^{-ik_\rho\rho\cos(\phi-\phi_k)}=\sum_{n=-\infty}^{\infty}(-i)^n J_n(k_\rho\rho)e^{-in(\phi-\phi_k)}, \quad (23)$$

$$e^{ik_\rho\rho'\cos(\phi'-\phi_k)}=\sum_{m=-\infty}^{\infty}(i)^m J_m(k_\rho\rho')e^{im(\phi'-\phi_k)}, \quad (24)$$

after cumbersome mathematical manipulation by properly grouping the terms involving the integrals for the ϕ_k variable and introducing the cylindrical vector wave functions, we end up with

$$\begin{aligned}\overline{\mathbf{G}}_{mm}(\mathbf{r},\mathbf{r}')=&\frac{i}{8\pi}\int_0^\infty dk_\rho\sum_{q=1}^{2}\frac{1}{M_q^2}\\&\times\sum_{n=-\infty}^{\infty}(-1)^n[a_q^z(k_\rho,k_{zq})\mathbf{M}_n^{(1)}(k_\rho,k_{zq})\\&+b_q^z(k_\rho,k_{zq})\mathbf{N}_n^{(1)}(k_\rho,k_{zq})+c_q^z(k_\rho,k_{zq})\\&\times\mathbf{L}_n^{(1)}(k_\rho,k_{zq})][a_q^{z'}(k_\rho,k_{zq})\mathbf{M}_{-n}^{(1)'}(k_\rho,-k_{zq})\\&+b_q^{z'}(k_\rho,k_{zq})\mathbf{N}_{-n}^{(1)'}(k_\rho,-k_{zq})\\&+c_q^{z'}(k_\rho,k_{zq})\mathbf{L}_{-n}^{(1)'}(k_\rho,-k_{zq})],\end{aligned} \quad (25)$$

where the primes over the vector wave functions denote that they are evaluated at $\mathbf{r}'$. Here, the techniques of mathematical manipulation are similar to those we have used in [9–12] to obtain the field representations in the source-free region. The expansion coefficients are found to be

$$a_q^\sigma(k_\rho,k_z)=-\frac{2iB_q^\sigma(k_\rho,k_z)}{k_\rho}, \quad (26)$$

$$b_q^\sigma(k_\rho,k_z)=-\frac{2k_qA_q^\sigma(k_\rho,k_z)}{k_\rho k_z}+\frac{2}{k_q}\left[1+\frac{k_\rho A_q^\sigma(k_\rho,k_z)}{k_z}\right], \quad (27)$$

$$c_q^\sigma(k_\rho,k_z)=\frac{2ik_z}{k_q^2}\left[1+\frac{k_\rho A_q^\sigma(k_\rho,k_z)}{k_z}\right], \quad (28)$$

with $k_q=(k_z^2+k_\rho^2)^{1/2}$, $\sigma=z$, and $k_z=k_{zq}$. $a_q^{\sigma'}(k_\rho,k_z)$, $b_q^{\sigma'}(k_\rho,k_z)$, and $c_q^{\sigma'}(k_\rho,k_z)$ can be separately derived from $a_q^\sigma(k_\rho,k_z)$, $b_q^\sigma(k_\rho,k_z)$, and $c_q^\sigma(k_\rho,k_z)$, with the replacement of $A_q^\sigma(k_\rho,k_z)$ and $B_q^\sigma(k_\rho,k_z)$ by their complex conjugates, respectively. The roots of the characteristic equation (7), $k_z=k_{zq}$, are such chosen that $\mathrm{Re}[k_{zq}]>0$ for $z>z'$, and $\mathrm{Re}[k_{zq}]<0$ for $z<z'$, where Re[ ] denotes the real part of the complex function. The cylindrical vector wave functions are defined as

$$\mathbf{M}_n^{(j)}(k_\rho,k_z)=\boldsymbol{\nabla}[\Psi_n^{(j)}(k_\rho,k_z)\mathbf{e}_z], \quad (29)$$

$$\mathbf{N}_n^{(j)}(k_\rho,k_z)=\frac{1}{k_q}\boldsymbol{\nabla}\times\mathbf{M}_n^{(j)}(k_\rho,k_z), \quad (30)$$

$$\mathbf{L}_n^{(j)}(k_\rho,k_z)=\boldsymbol{\nabla}\Psi_n^{(j)}(k_\rho,k_z), \quad (31)$$

where the generating function is

$$\Psi_n^{(j)}(k_\rho,k_z)=Z_n^{(j)}(k_\rho\rho)e^{-i(k_zz+n\phi)}, \quad (32)$$

and

$$Z_n^{(j)}(k_\rho\rho)=\begin{cases}J_n(k_\rho\rho), & j=1\\ Y_n(K_\rho\rho), & j=2\\ H_n^{(1)}(k_\rho\rho), & j=3\\ H_n^{(2)}(k_\rho\rho), & j=4.\end{cases} \quad (33)$$

The Green dyadic of electric type $\overline{\mathbf{G}}_{ee}(\mathbf{r},\mathbf{r}')$ can be obtained from $\overline{\mathbf{G}}_{mm}(\mathbf{r},\mathbf{r}')$, with the replacement of a_q^z, b_q^z, c_q^z, $a_q^{z'}$, $b_q^{z'}$, and $c_q^{z'}$ by d_q^z, e_q^z, f_q^z, $d_q^{z'}$, $e_q^{z'}$, and $f_q^{z'}$, respectively. Here the expansion coefficients are determined as

$$d_q^\sigma(k_\rho,k_z)=-\frac{2iD_q^\sigma(k_\rho,k_z)}{k_\rho}, \quad (34)$$

$$e_q^\sigma(k_\rho,k_z)=-\frac{2k_zC_q^\sigma(k_\rho,k_z)}{k_\rho k_q}+\frac{2[ik_0\gamma-k_\rho B_q^\sigma(k_\rho,k_z)]}{k_q\omega\varepsilon_z}, \quad (35)$$

$$f_q^\sigma(k_\rho,k_z)=\frac{2i}{k_q^2}\left[k_\rho C_q^\sigma(k_\rho,k_z)+\frac{k_z[ik_0\gamma-k_\rho B_q^\sigma(k_\rho,k_z)]}{\omega\varepsilon_z}\right] \quad (36)$$

for $\sigma=z$ and $k_z=k_{zq}$. $d_q^{\sigma'}(k_\rho,k_z)$, $e_q^{\sigma'}(k_\rho,k_z)$, and $f_q^{\sigma'}(k_\rho,k_z)$ can be separately obtained from $d_q^\sigma(k_\rho,k_z)$, $e_q^\sigma(k_\rho,k_z)$, and $f_q^\sigma(k_\rho,k_z)$, with the replacement of $C_q^\sigma(k_\rho,k_z)$, $D_q^\sigma(k_\rho,k_z)$, and $[ik_0\gamma-k_\rho B_q^\sigma(k_\rho,k_z)]$ by their complex conjugates, respectively. The pseudotype Green dyadics $\mathbf{G}_{em}(\mathbf{r},\mathbf{r}')$ can be obtained from $\mathbf{G}_{mm}(\mathbf{r},\mathbf{r}')$, with the substitution of a_q^z, b_q^z, and c_q^z by d_q^z, e_q^z, and f_q^z, respectively; and $\mathbf{G}_{me}(\mathbf{r},\mathbf{r}')$ can be derived from $\mathbf{G}_{mm}(\mathbf{r},\mathbf{r}')$, with the replacement of $a_q^{z'}$, $b_q^{z'}$, and $c_q^{z'}$ by $d_q^{z'}$, $e_q^{z'}$, and $f_q^{z'}$, separately.

It should be mentioned that the present eigenfunction expansion of the Green dyadics can be reduced to the counterparts of a reciprocal chiral medium [20], if letting $\varepsilon_t=\varepsilon_z=\varepsilon$, $\mu_t=\mu_z=\mu$, $\alpha=\gamma=\xi_c$, and $\beta=0$ in the constitutive relations. This set of eigenfunction representations of the Green dyadics can be used to construct the Green dyadics of planarly multilayered reciprocal uniaxial bianisotropic media, by applying the method of scattering superposition [4,20] and appropriate electromagnetic boundary conditions.

Straightforward mathematical analysis reveals that for dipole sources parallel to the z axis, only terms corresponding to $n=0$ exist for the Green dyadics, while the Green dyadics of dipole sources perpendicular to the z axis contain only the $n=1$ terms. Therefore, Sommerfeld-Weyl-type integrals of dipole radiation in a reciprocal uniaxial bianisotropic medium involve only those Sommerfeld-Weyl-type integrals of dipole radiation in an isotropic medium [35]. So various approximate, asymptotic, and numerical methods for Sommerfeld-Weyl-type integrals [35] can be applied to study the electromagnetic resonance, radiation, propagation, and scattering phenomena of planarly multilayered reciprocal uniaxial bianisotropic media.

B. ANALYTICAL EVALUATION OF THE INTEGRAL WITH RESPECT TO THE SPECTRAL RADIAL WAVE NUMBER

In this subsection, we will try to represent Eq. (21) in the form of the eigenfunction expansion in terms of the cylindrical vector wave functions. To this end, employing identities (23) and (24), the integral with respect to the spectral radial wave number k_ρ is analytically evaluated by applying the residue calculus through a modified contour in the k_ρ plane, which results in

$$\bar{\mathbf{G}}(\mathbf{r},\mathbf{r}')=\begin{bmatrix}\bar{\mathbf{G}}_{ee}(\mathbf{r},\mathbf{r}') & \bar{\mathbf{G}}_{em}(\mathbf{r},\mathbf{r}')\\ \bar{\mathbf{G}}_{me}(\mathbf{r},\mathbf{r}') & \bar{\mathbf{G}}_{mm}(\mathbf{r},\mathbf{r}')\end{bmatrix}=\begin{cases}\dfrac{i}{16\pi^2}\displaystyle\int_{-\infty}^{\infty}dk_z\int_{\phi_k=0}^{2\pi}d\phi_k\sum_q\frac{\mathbf{\Psi}_q^\rho(\mathbf{k})\mathbf{\Psi}_q^{\rho*}(\mathbf{k})}{N_q^2}e^{-ik_z(z-z')}\\ \times\displaystyle\sum_{n=-\infty}^{\infty}(-i)^nJ_n(k_{\rho q}\rho)e^{-in(\phi-\phi_k)}\sum_{m=-\infty}^{\infty}i^mH_m^{(2)}(k_{\rho q}\rho')e^{im(\phi'-\phi_k)}, & \rho\leqslant\rho'\\ \dfrac{i}{16\pi^2}\displaystyle\int_{-\infty}^{\infty}dk_z\int_{\phi_k=0}^{2\pi}d\phi_k\sum_q\frac{\mathbf{\Psi}_q^\rho(\mathbf{k})\mathbf{\Psi}_q^{\rho*}(\mathbf{k})}{N_q^2}e^{-ik_z(z-z')}\\ \times\displaystyle\sum_{n=-\infty}^{\infty}(-i)^nH_n^{(2)}(k_{\rho q}\rho)e^{-in(\phi-\phi_k)}\sum_{m=-\infty}^{\infty}i^mJ_m(k_{\rho q}\rho')e^{im(\phi'-\phi_k)}, & \rho\geqslant\rho'.\end{cases} \tag{37}$$

Here, we have employed the identity [4]

$$\int_0^\infty dk_\rho\frac{\bar{\mathbf{T}}[J_n(k_\rho\rho)J_n(k_\rho\rho')]}{(k_{\rho q}-k_\rho)N_q^2}=\frac{i\pi}{2N_q^2}\bar{\mathbf{T}}[H_n^{(2)}(k_{\rho q}\rho_>)J_n(k_{\rho q}\rho_>)], \tag{38}$$

where $\rho_>=\max(\rho,\rho')$, $\rho_<=\min(\rho,\rho')$, and $\bar{\mathbf{T}}$ stands for a spatial dyadic operator, having the property of $\bar{\mathbf{T}}(k_\rho)=-\bar{\mathbf{T}}(-k_\rho)$.

Substituting the explicit expression of $\mathbf{\Psi}_q^\rho(k)$ into (37), and properly grouping the terms involving the integrals with respect to the ϕ_k variable, the Green dyadics in the reciprocal uniaxial bianisotropic medium can be represented in terms of the cylindrical vector wave functions:

$$\begin{aligned}\bar{\mathbf{G}}_{mm}(\mathbf{r},\mathbf{r}')=&\frac{i}{16\pi}\int_{-\infty}^{\infty}dk_z\sum_{q=1}^{2}\frac{1}{N_q^2}\\&\times\sum_{n=-\infty}^{\infty}(-1)^n[a_q^\rho(k_{\rho q},k_z)\mathbf{M}_n^{(\tau_1)}(k_{\rho q},k_z)\\&+b_q^\rho(k_{\rho q},k_z)\mathbf{N}_n^{(\tau_1)}(k_{\rho q},k_z)+c_q^\rho(k_{\rho q},k_z)\\&\times\mathbf{L}_n^{(\tau_1)}(k_{\rho q},k_z)][a_q^{\rho'}(k_{\rho q},k_z)\mathbf{M}_{-n}^{(\tau_2)'}(k_{\rho q},-k_z)\\&+b_q^{\rho'}(k_{\rho q},k_z)\mathbf{N}_{-n}^{(\tau_2)'}(k_{\rho q},-k_z)\\&+c_q^{\rho'}(k_{\rho q},k_z)\mathbf{L}_{-n}^{(\tau_2)'}(k_{\rho q},-k_z)].\end{aligned} \tag{39}$$

$\bar{\mathbf{G}}_{ee}(\mathbf{r},\mathbf{r}')$ can be obtained from $\bar{\mathbf{G}}_{mm}(\mathbf{r},\mathbf{r}')$, with the replacement of a_q^ρ, b_q^ρ, c_q^ρ, $a_q^{\rho'}$, $b_q^{\rho'}$, and $c_q^{\rho'}$ by d_q^ρ, e_q^ρ, f_q^ρ, $d_q^{\rho'}$, $e_q^{\rho'}$, and $f_q^{\rho'}$, respectively; $\bar{\mathbf{G}}_{em}(\mathbf{r},\mathbf{r}')$ can be derived from $\mathbf{G}_{mm}(\mathbf{r},\mathbf{r}')$, with the separate substitution of a_q^ρ, b_q^ρ, and c_q^ρ by d_q^ρ, e_q^ρ, and f_q^ρ; and $\mathbf{G}_{me}(\mathbf{r},\mathbf{r}')$, can be obtained from $\mathbf{G}_{mm}(\mathbf{r},\mathbf{r}')$, with the replacement of $a_q^{\rho'}$, $b_q^{\rho'}$, and $c_q^{\rho'}$ by $d_q^{\rho'}$, $e_q^{\rho'}$, and $f_q^{\rho'}$, respectively. Here $\tau_1=1$ and $\tau_2=4$ for $\rho\leqslant\rho'$, and $\tau_1=4$ and $\tau_2=1$ for $\rho\geqslant\rho'$. The expansion coefficients used here can be straightforwardly obtained from Eqs. (26)–(28) and (34)–(36), with the substitution of $\sigma=\rho$ and $k_\rho=k_{\rho q}$.

In Eq. (39), $k_{\rho3}$ and $k_{\rho4}$ are not included in the summation since $k_{\rho3}=-k_{\rho1}$ and $k_{\rho4}=-k_{\rho2}$, and these symmetric roots are automatically taken into account as the spectral azimuthal angle ϕ_k spans from 0 to 2π. It should be pointed out that the Green dyadics represented in the forms of the eigenfunction expansion, as given in this subsection, can be verified by comparing their special forms with the counterparts of reciprocal chiral medium [21] and isotropic medium [4]. Moreover, they can be used to construct the Green dyadics of a cylindrically multilayered structure consisting of the reciprocal uniaxial bianisotropic media, by employing the method of scattering superposition [4,21] and appropriate electromagnetic boundary conditions.

The resulting equations in this subsection indicate that the electromagnetic waves in an unbounded reciprocal uniaxial bianisotropic medium are transversely outgoing for $\rho\geqslant\rho'$, and transversely standing for $\rho\leqslant\rho'$. This physical property of the electromagnetic waves is similar to that of a dielectric leaky antenna with an infinitely long circular cylindrical structure, positioned in an unbounded isotropic medium.

From the present formulations, it is easily seen that $\overline{\mathbf{G}}_{ee}(\mathbf{r},\mathbf{r}')=\overline{\mathbf{G}}_{ee}^{T}(\mathbf{r}',\mathbf{r})$, $\overline{\mathbf{G}}_{mm}(\mathbf{r},\mathbf{r}')=\overline{\mathbf{G}}_{mm}^{T}(\mathbf{r}',\mathbf{r})$, and $\overline{\mathbf{G}}_{em}(\mathbf{r},\mathbf{r}')=-\overline{\mathbf{G}}_{me}^{T}(\mathbf{r}',\mathbf{r})$, which can also be directly obtained from the reciprocal theorem [22]. In addition, it can be straightforward to derive the mathematical relationship among these Green dyadics, which could also be obtained from the definition of the Green dyadics (19) and the source-incorporated Maxwell's equations (3):

$$\overline{\mathbf{G}}_{em}(\mathbf{r},\mathbf{r}')=-\frac{i}{\omega}\,\overline{\boldsymbol{\varepsilon}}^{-1}\cdot[(\nabla\times\overline{\mathbf{I}}-i\omega\overline{\boldsymbol{\xi}})\cdot\overline{\mathbf{G}}_{mm}(\mathbf{r},\mathbf{r}')], \tag{40}$$

$$\overline{\mathbf{G}}_{me}(\mathbf{r},\mathbf{r}')=\frac{i}{\omega}\,\overline{\boldsymbol{\mu}}^{-1}\cdot[(\nabla\times\overline{\mathbf{I}}+i\omega\overline{\boldsymbol{\zeta}})\cdot\overline{\mathbf{G}}_{ee}(\mathbf{r},\mathbf{r}')]. \tag{41}$$

Here $\overline{\mathbf{I}}$ denotes the 3×3 unit dyadic.

The electromagnetic fields associated with the exciting sources can be obtained from (19), by substituting either set of the above-presented Green dyadics. From the present formulations, it can be seen that the solutions of the source-incorporated Maxwell's equations for a homogeneous reciprocal uniaxial bianisotropic medium are composed of two eigenwaves traveling with different wave numbers, and each of these eigenwaves is a superposition of two transverse waves (**M** and **N** represent two transverse waves) and a longitudinal wave.

The essential idea of the method employed here, which is standard and straightforward, can be exploited to derive the eigenfunction expansion of the Green dyadics in the spherical coordinate system. However, since the wave numbers of the eigenwaves are functions of the direction of these eigenwaves, simple compact forms of the field representations (corresponding to those of [9–12]) in the source-free reciprocal uniaxial bianisotropic media by the spherical vector wave functions cannot be obtained, and the solutions of the source-incorporated Maxwell's equations cannot be directly formulated in compact forms of the spherical vector wave functions, either. In the circular cylindrical coordinate system, however, it is seen from the present formulations that, since the wave numbers of the eigenwaves do not depend on the spectral azimuthal angle ϕ_k, the solutions of the source-incorporated Maxwell's equations in the reciprocal uniaxial bianisotropic medium can be represented in compact forms of the cylindrical vector wave functions of isotropic media.

IV. CONCLUDING REMARKS

In the present paper the eigenfunction expansion of the Green dyadics in an unbounded reciprocal uniaxial bianisotropic medium are developed in terms of the cylindrical vector wave functions, based on the concept of spectral eigenwaves. The analysis indicates that the solutions of the source-incorporated Maxwell's equations in a reciprocal uniaxial bianisotropic medium are composed of two eigenwaves traveling with different wavenumbers, and each of these eigenwaves is a superposition of two transverse waves and a longitudinal wave. The Green dyadics for planarly and cylindrically multilayered structures consisting of the reciprocal uniaxial bianisotropic media can be obtained straightforwardly by employing the methods of scattering superposition and appropriate electromagnetic boundary conditions, respectively. The constraint condition of the present approach, which is standard and straightforward, is that the spectral longitudinal (and radial) wave numbers do not depend on the spectral azimuthal angle ϕ_k. In spite of this constraint condition, which makes the approach employed here only applicable to a limited class of materials, the present formulations can be used to analyze and understand the physical phenomena of the source-incorporated electromagnetic boundary value problems involving unbounded or multilayered reciprocal uniaxial bianisotropic media. It is of interest to note that the cylindrical vector wave functions can be expanded as discrete sums of the spherical vector wave functions [36]; therefore the present formulations could be extended to solve the problems of spherical structures. Since the reciprocal uniaxial bianisotropic media studied here recover the isotropic media [4], uniaxial bianisotropic media [26], transversely chiral uniaxial bianisotropic media [24], uniaxial chiro-omega media [27], and the extensively studied chiral media [20,21], the present formulations can be specifically applied to these materials, and theoretically verified by comparing their special forms with the already existed results corresponding to the isotropic media [4] and reciprocal chiral media [20,21]. When the present reciprocal uniaxial bianisotropic media reduce to the media treated in [30,33], the Green dyadics formulated here can be represented in simple closed forms such as those of [30–33], after the integrals are explicitly evaluated, respectively. In addition, the method employed here can be extended to derive the eigenfunction expansion of Green dyadics in other kinds of media, such as transversely isotropic elastic media [37], transversely isotropic piezoelectric solids [38], and transversely isotropic saturated porous media [39]. Although the present formulations are somewhat cumbersome, which is inevitable due to the complexity of the material we have tried to tackle, they are important and useful in analyzing and understanding the (equivalently) source-incorporated electromagnetic phenomena of the reciprocal uniaxial bianisotropic media. Even if there exist two operations in the present formulations which are over infinite domains, the convergence of these operations has been numerically examined for the source-free problems [9–12]. For the source-incorporated problems, verification for the convergence of the operations is straightforward. Moreover, these two operations over infinite domains also exist for isotropic media [4] and reciprocal chiral media [20,21]; therefore various numerical and asymptotic methods [35] can be employed to simplify the computation in practical applications. In our previous investigation, the problems we treated are the source-free Maxwell's equations in the given complex media [9–12], while in the present study we have tried to solve analytically the source-incorporated problems by the cylindrical vector wave functions. From a mathematical point of view, our previous investigation [9–12] is essentially based on the method of spectral angular expansion, while the present starting point is the completeness property of the spectral eigenwaves. It is believed that the present formulations provide a fundamental basis to analyze the (equivalently) source-incorporated electromagnetic phenomena of the reciprocal uniaxial bianisotropic media, to study the Raman and fluorescent scattering by active molecules embedded in a reciprocal uniaxial bianisotropic medium, and to understand and interpret the physical process of this class of media. Applications of the present formulations in analyzing the electromagnetic scattering, propagation, resonance, and radiation phenomena relevant to the reciprocal uniaxial bianisotropic media are under investigation, and will be reported on in subsequent manuscripts.

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